MATH 251 – Multivariate and Vector Calculus

Course Description from Bulletin: Analytic geometry in three-dimensional space. Partial derivatives. Multiple integrals. Vector analysis. Applications. (4-0-4)

Enrollment: Required for AM majors and some engineering majors

Textbook(s): James Stewart, *Calculus* (9th Ed.), Cengage (2021), ISBN:9781337624183 (Recommended if the entire Calculus sequence will be taken. For MATH 251 only, Stewart's *Multivariable Calculus* suffices.)

Other required material: WebAssign access (comes bundled with Stewart Calculus)

Prerequisites: Math 152

Objectives:

1. Students will learn to solve problems in three-dimensional space by utilizing vectors and vector-algebraic concepts. This includes representation in Cartesian, cylindrical and spherical coordinates.
2. Students will be able to describe the path, velocity and acceleration of a moving body in terms of vector-valued functions, and to apply the derivative and integral operators on space curves in order to characterize the length, curvature and torsion of a smooth curve.
3. Students will learn to extend the notion of continuity and differentiability to functions of several variables, and be able to interpret partial and directional derivatives as rates of change.
4. Students will be able to use partial differentiation to solve optimization problems. This includes being able to solve constrained optimization problems via Lagrange multipliers.
5. Students will learn to extend the notion of a definite integral from a one-dimensional to an *n*-dimensional space, and be able to describe and evaluate double and triple integrals in Cartesian and curvilinear coordinates.
6. Students will be able to work with vector-valued functions of several variables (i.e., vector fields) and be able to compute line and surface integrals.
7. Students will be able to use the theorems of Green, Stokes, and Gauss to solve classical physics problems.

Lecture schedule: Three 75 minute lectures per week

Course Outline:	Hours
1. Vectors and the Geometry of Space	10
a. Vectors in the plane	
b. Cartesian coordinates and vectors in space	
c. Dot products and cross products	
d. Lines and planes in space	
e. Cylinders and quadric surfaces	
f. Cylindrical and spherical coordinates	
2. Vector Functions and their Derivatives	6
a. Vector-valued functions and motion in space	
b. Space curves	

c. Arc length and the unit tangent vector

3. Partial Derivatives 12
    a. Functions of several variables
    b. Limits and continuity, partial derivatives, differentiability
    c. Linearization and differentials
    d. Chain rule
    e. Gradient vector, tangent planes, directional derivatives
    f. Extreme values and saddle points,
    g. Lagrange multipliers
    h. Taylor's formula
4. Multiple Integrals 13
    a. Double integrals
    b. Areas, moments, and centers of mass
    c. Double integrals in polar form
    d. Triple integrals in rectangular coordinates
    e. Masses and moments in 3-D
    f. Triple integrals in cylindrical and spherical coordinates
    g. Substitutions in multiple integrals
5. Vector Calculus 13
    a. Integration in vector fields
    b. Line integrals
    c. Vector fields
    d. Work, circulation, and flux
    e. Path independence, potential functions, and conservative fields
    f. Green's theorem in the plane
    g. Surface area and surface integrals
    h. Parameterized surfaces
    i. Stokes' theorem
    j. Divergence theorem and a unified theory

Assessment:		
	Homework/Quizzes	35%
	3 Tests	30%
	Common Final Exam	35%

Syllabus prepared by: Andre Adler and Greg Fasshauer
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